A Model for the Expected Running Time of Collision Detection using AABB Trees

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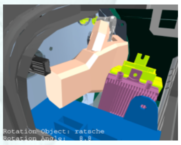
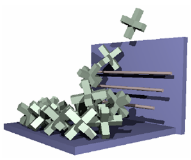
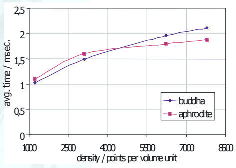
Motivation

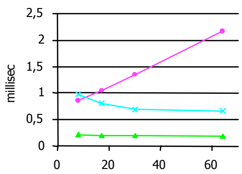
- Collision Detection is ubiquitous in VR and many physically-based simulation apps






- Obviously: worst-case running time is in $O(n^2)$
- But, we all have seen real-world running time behavior like this:





Goal

- Gain (theoretical) understanding of experienced running times
- Utilize to optimize collision detection
- Better heuristics for probabilistic collision detection

Related Work

- Distance of convex polytopes [Dobkin & Kirkpatrick, 1985]:

  $O(\log^2 n)$, $n =$ number of faces

- Distance of convex polytopes [Lin & Canny, 1991]:

  $O(\sqrt{n})$, worst-case

  $O(1)$, expected time, bounded rotation

- General polytopes, fixed trajectory [Schömer & Thiel, 1995]:

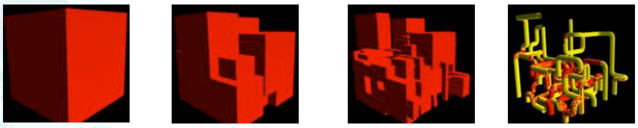
  $O(n^{\frac{5}{3}+\varepsilon})$

- Alll intersections of n convex polytopes [Suri et al., 1998]:

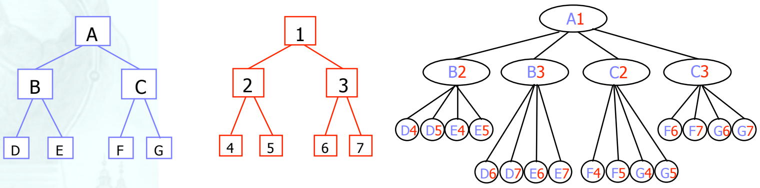
  $O((n+k)\log^2 n)$, $k =$ # intersecting pairs

Hierarchical CD

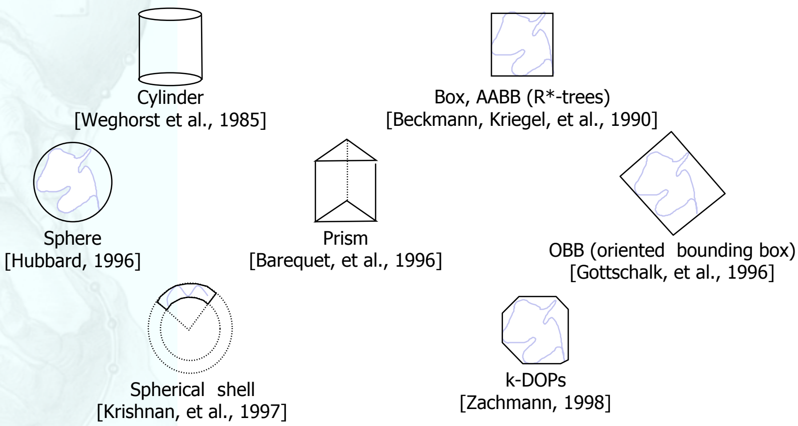
- Hierarchical CD is most common technique for rigid bodies
- BV hierarchy (BVH) is constructed in preprocessing:



- Simultaneous traversal of two BVHs = single traversal of one BV test tree (BVTT)



- Lots of different BVs have been proposed, e.g.:



- In the following: use AABBs

- The Cost formula [Weghorst et al. 1984; Gottschalk et al. 1996]:

$$T = N_v C_v + N_p C_p + N_u C_u + C_i$$

$N_v, C_v =$ num., costs of BV overlap test, resp.
$N_p, C_p =$ num., costs of primitive intersection test
$N_u, C_u =$ num., costs of BV update, resp.
$C_i =$ initialization costs

- Obviously: $T(n) \sim N_v(n)$

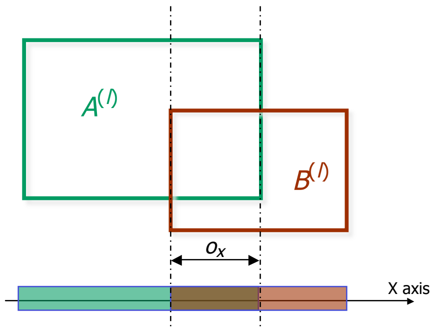
- Goal: determine $E[\ N_v(n)\ ] = \tilde{N}_v(n)$
  = number of nodes in the BVTT that are visited on average

The Model to Determine $\tilde{N}_v(n)$

- Assumption: use AABBs
- Estimate probability of BV overlap on some level l
- Yields product of conditional probabilities
- Estimate conditional probability by geometric reasoning

Terminology

- $P[\, A^{(l)} \cap B^{(l)} \neq \varnothing \,]$ = probability that two AABBs on level l overlap each other
- In the following, just write $P[\, A^{(l)} \cap B^{(l)} \,]$
- X-Overlap $o_x :=$ length of overlap of slabs of AABBs



The Chain of Probabilities

- Obviously, the expected total number of BV overlaps is

$$\tilde{N}_V(n) = \sum_{l=1}^{d} \tilde{N}_V^{(l)} = \sum_{l=1}^{d} 4^l P[\, A^{(l)} \cap B^{(l)} \,] \quad (1)$$

- Recall that $X \subseteq Y \;\Rightarrow\; P[X] = P[Y] \cdot P[\, X \mid Y \,]$
- Turn $P[\, A^{(l)} \cap B^{(l)} \,]$ into conditional probability that "defers" the probability up one level in the hierarchy:

$$P[\, A^{(l)} \cap B^{(l)} \,] = P[\, A^{(l)} \cap B^{(l)} \mid A^{(l-1)} \cap B^{(l-1)} \wedge o_x^{(l)} > 0 \,] \cdot P[\, A^{(l-1)} \cap B^{(l-1)} \wedge o_x^{(l)} > 0 \,]$$

- Resolve further:

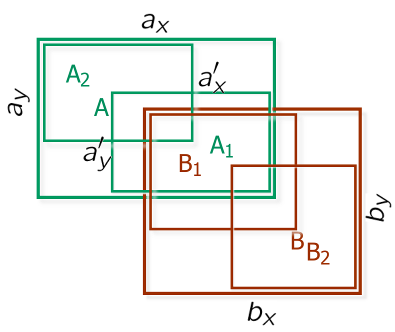
$$P[\,A^{(l)} \cap B^{(l)}\,] = P[\,A^{(l)} \cap B^{(l)} \mid A^{(l-1)} \cap B^{(l-1)} \wedge o_x^{(l)} > 0\,] \cdot P[\,A^{(l-1)} \cap B^{(l-1)}\,] \cdot P[\,o_x^{(l)} > 0 \mid A^{(l-1)} \cap B^{(l-1)}\,]$$

- "Unroll" recurrence:

$$P[\,A^{(l)} \cap B^{(l)}\,] = \prod_{i=1}^{l} P[\,A^{(i)} \cap B^{(i)} \mid A^{(i-1)} \cap B^{(i-1)} \wedge o_x^{(i)} > 0\,] \cdot \prod_{i=1}^{l} P[\,o_x^{(i)} > 0 \mid A^{(i-1)} \cap B^{(i-1)}\,]$$

The Geometric Probability

- Goal: estimate $P[\,A^{(l)} \cap B^{(l)} \mid A^{(l-1)} \cap B^{(l-1)} \wedge o_x^{(l)} > 0\,]$
- Terminology:
  - Denote parent boxes by A, B
  - Denote extents by a_x, a_y, b_x, b_y
  - Denote child boxes by A_1, A_2, B_1, B_2
  - Denote child box extents by $a'_x, a'_y, \ldots$

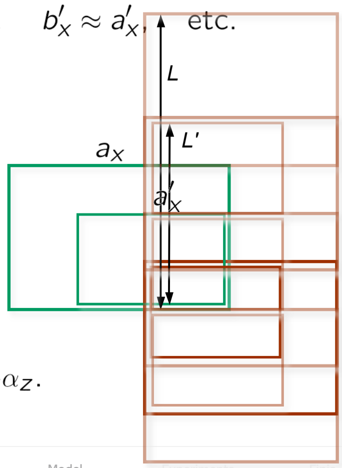


- Re-stated goal: estimate

$$p_{ij} := P[\,A_i \cap B_j \mid A \cap B \wedge o_x > 0]$$

- Assumptions for now:
  - *BV diminishing factor* : $a'_x = \alpha_x a_x, \quad a'_y = \alpha_y a_y, \quad$ etc.
  - BVs on same scale, i.e.: $b_x \approx a_x, \quad b'_x \approx a'_x, \quad$ etc.
- Look at p_{11} first:
  - Preconditions:
    - x-overlap $o_x > 0$, and
    - parent boxes overlap
  - Probability:

$$p_{11} = \frac{\text{area}(L')}{\text{area}(L)} = \ldots = \alpha_y \alpha_z .$$



- Good news:

$$p_{22} = p_{12} = p_{21} = \alpha_y \alpha_z$$

- By analogous reasoning, we get:

$$P[\, o_x^{(l)} > 0 \mid A^{(l-1)} \cap B^{(l-1)} \,] \approx \alpha_x$$

- Plug all this into Equation (1):

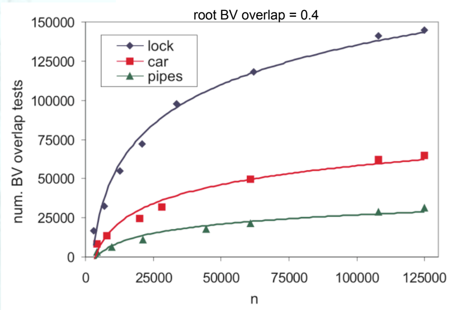
$$\tilde{N}_V(n) \leq \sum_{l=1}^{d} (4\alpha_x\alpha_y\alpha_z)^l \in O\left(n^{\lg(4\alpha_x\alpha_y\alpha_z)}\right)$$

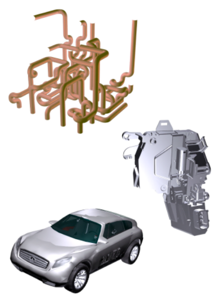
- Effect of diminishing factor α :

α	$T(n)$
$1/4$	$O(\lg n)$
≈ 0.35	$O(\sqrt{n})$
$3/4$	$O(n^{1.58})$

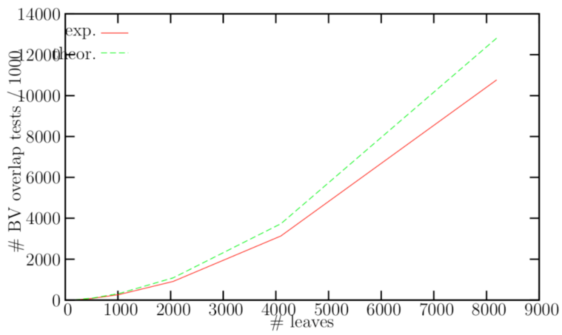
Experiments

- Experiment:
  - Construct simple AABB over CAD objects
  - Count number of nodes in BVTT visited by simultaneous traversal



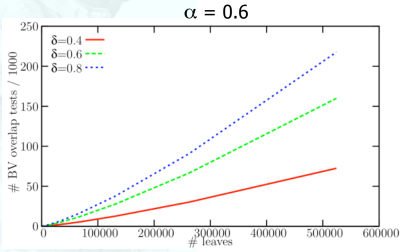


- Experiments utilizing artificial BVHs:
  - Controlled "layout" and diminishing factor α
- Experimental versus theoretical estimates:

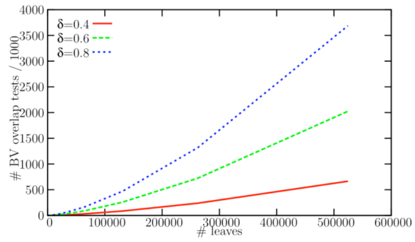


- Interdependence between α and root BV overlap δ :

α = 0.6



α = 0.7



Application

- Time-critical collision detection
- Probabilistic collision detection:
  - Store average α at root of every sub-tree
  - Estimate # BV overlap tests using our model
  - Prioritize traversal based on this number

Conclusions

- Average-case analysis of simultaneous traversal of AABB trees
- New model to estimate the average running time
- Experiments to support correctness of our model

Future Work

- Improve model:
  - variable BV diminishing factor (probably easy)
  - integrate root BV overlap into model
- Consider other BV types (possibly hard)
- Utilize for probabilistic collision detection
- Derive method for average-case analysis of running time for concrete BVHs

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Introduction RelatedWork Overview Model Experiments Finis